On the Formation and Primitive Structure of the Solar System.

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The development of any branch of science is generally a slow and gradual process. The obvious truths which suggested to Laplace his celebrated hypothesis of the solar system had been for ages well known to astronomers ; but, as in the case of the earlier geological observations, they had been regarded, without any just reason, as *ultimate facts.* So now we have numerous results of observation in regard to the rings of Saturn, the zone of asteroids, the relative distances of the planets, &c., the study of which, it is believed, may lead to new and important discoveries. "These hieroglyphics older than the Nile," pointing back to the epochs at which the planets were born, will doubtless in the future be more or less clearly deciphered, and the ancient history of the solar system at least partially developed.

It is a very remarkable fact in regard to the systems of both primary and secondary planets that the periods, without any exception, have very simple relations of approximate commensurability. This truth, though obvious on mere inspection, seems not to have attracted the special notice of astronomers, as no attempt had been made, previous to that of the writer, to assign its physical cause. A general view of these approximations is presented in the following tables, where the periods of the primary planets, Mercury, Venus, &c., are represented by P^{I}, P^{II}, &c., and those of the satellites by p^{I}, p^{II}, &c.

I.

THE PRIMARY SYSTEM.

$\frac{1}{2}P^{VIII}$	$=82.37$ years	$=P^{VII}$	-1.65 y.
$\frac{1}{3}P^{VII}$	$=28.01$ "	$=P^{VI}$	-1.45
$\frac{2}{5}P^{VI}$	$=11.78$ "	$=P^{V}$	-0.08
$\frac{1}{6}P^{V}$	$=1.97$ "	$=P^{IV}$	$+0.09$
$\frac{1}{12}P^{V}$	$=0.988$ "	$=P^{III}$	-0.012
$\frac{2}{3}P^{III}$	$=0.667$ "	$=P^{II}$	$+0.052$
$\frac{2}{5}P^{II}$	$=0.246$ "	$=P^{I}$	-0.005

II.

THE JOVIAN SYSTEM.

$\frac{3}{7}p^{IV}$	$=p^{III}$	-3^{m}	9	s
$\frac{1}{2}p^{III}$	$=p^{II}$	$+36$	40	
$\frac{1}{2}p^{II}$	$=p^{I}$	$+9$	45	

III.

THE SATURNIAN SYSTEM.

$$\tfrac{2}{7}p^{VIII} = p^{VII} + (?)$$

$$\tfrac{1}{5}p^{VIII} = p^{VI} - 1^h\ 54^m\ 41^s$$

$$\tfrac{2}{7}p^{VI} = p^{V} + 0\ \ 55\ \ 13$$

$$\tfrac{1}{6}p^{VI} = p^{IV} - 1\ \ 54\ \ 15$$

$$\tfrac{1}{2}p^{IV} = p^{II} - 0\ \ 2\ \ 33$$

$$\tfrac{1}{2}p^{III} = p^{I} + 0\ \ 1\ \ 50$$

IV.

THE URANIAN SYSTEM.

$$\tfrac{2}{3}p^{IV} = p^{III} + 6^h$$

$$\tfrac{1}{2}p^{III} = p^{II} + 5$$

$$\tfrac{3}{5}p^{II} = p^{I} - 1$$

It is infinitely improbable that all these coincidences should be purely accidental. Their physical cause is a legitimate object of research, and the writer is vain enough to believe that he has suggested the true one.* Before proceeding with our discussion, however, it may be proper to indicate such modifications of the nebular hypothesis as seem to be demanded by recent discoveries.

The view generally received in regard to the formation of the solar system has been that equatorial rings were abandoned *only* in the vicinity of the present planetary orbits. As the writer has elsewhere observed, however, "it seems highly probable that, after first reaching the point at which gravity was counterbalanced by the centrifugal force arising from the rotation of the contracting spheroid, a continuous succession of narrow rings would be thrown off in close proximity to each other, and revolving in different periods according to Kepler's third law." But in this matter we are not left to mere speculation. The zone of minor planets has evidently not been produced by a single annulus, all the parts of which had, at first, nearly equal velocities. On the contrary, it must have resulted from an almost continuous abandonment of narrow rings, from the exterior limit at the mean distance 3.50, down to the interior, at 2.20. The rings of Saturn, moreover, afford a similar index to the process of planetary formation.

Let us assume, then, the existence of a central mass S, with a ring R, and an exterior planet P. The particles of the ring having different distances from the centre of motion will move with different velocities. Let

*Met. Astr., Ch. XIII., and Monthly Notices of the R. A. S., vol. XXIX.

S p be the distance at which a planetary molecule would revolve in one half the period of the planet P. The disturbing effect of P will render the orbit of p more and more eccentric. The particle, therefore, must be brought into contact, either in aphelion or perihelion, with other parts of the ring, thus forming a planetary nucleus at such distance that its period would be *nearly* one-half that of the exterior planet. Similar reasoning will apply to the distances at which the ratio of the periods would be $\frac{1}{3}$, $\frac{2}{5}$, or any simple relation of commensurability. We have thus an extremely simple explanation of the facts embodied in the preceding tables. Should it be objected that this theory fails to account for the formation of the most remote planet, it may be answered that the first separation of matter from the condensing nebula probably occurred before the mass had assumed a symmetrical form. The successive ratios of the periods from Neptune to Jupiter are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{2}{5}$. With Jupiter, "the giant of the solar system," the process of planet formation seems to have culminated ; the mass of this stupendous globe being nearly three times greater than that of all other members of the solar family united. But why have we no planet of any considerable magnitude whose period is one-half, one-third, or two-fifths that of Jupiter? It may be answered, in the first place, that the matter of asteroid ring was so extremely rare that the intersection of orbits failed to produce large planetary nuclei. The question recurs, however, whence the *small mass* of the ring immediately interior to the *largest* member of the system? The circumstances of the primitive asteroid-ring were different from those of any other. As its successive portions were thrown off at the equator of the solar nebula they would be liable to great perturbations by Jupiter. The perihelion distance of portions of the zone might thus become less than the equatorial radius of the spheroid by which they had been abandoned. A considerable proportion of the matter originally separated may have been thus re-united to the parent mass.*

The writer has shown however, that in the distribution of the mean distances of the asteroids, we have indications of an order similar to that of the exterior planets. This fact is rendered still more conspicuous by recent discoveries. The distances at which the periods of asteroids would be one-half, two-fifths, and one-third that of Jupiter, are respectively, 3.2776, 2.8245, and 2.5012. Between the mean distances 3.22 and 3.32 no asteroid has yet been discovered ; while between 3.12 and 3.22 there are no less than 12. Between 2.78 and 2.88, the interval containing the distance at which five times the period of a planet would be equal to twice that of Jupiter, only two have been detected ; while in the equal space immediately interior, from 2.68 to 2.78, there are 21. Finally, between 2.45 and 2.55, the space in the middle of which an asteroid's period would be one-third that of Jupiter, the number of known asteroids is 4 ; while in the equal space immediately interior there are 20, and in that exterior, 15. These facts are certainly very remarkable, and deserve the earnest consideration of astronomers.

*See Proc. Am. Phil. Soc., Aug. 19, 1870.

The preceding table of the primary system seems to indicate the dependence of the periods of Mars and the earth on the powerful mass of Jupiter. The relations expressed between the periods of the earth, Venus, and Mercury are sufficiently obvious. It is worthy of remark that the *original* distances of the exterior planets have been, in all probability, sensibly diminished. While the solar nebula was undergoing the process of condensation all cometary and meteoric matter attracted towards its centre, would, if the perihelion distance were considerably less than the radius of the nebula, become incorporated with the central body. This growth of the solar mass would produce a shortening of the periodic times of all planets previously formed.

The approximations to commensurability in the secondary systems are still more striking, and must produce the impression in every inquiring mind that they are not without their physical significance.

The rings of Saturn formerly supposed to be solid and continuous, are now regarded as consisting of an indefinite number of extremely small satellites. They are, in short, a compact cluster of secondary asteroids, analogous to the primary zone between Mars and Jupiter. In the latter, it is true, a large proportion of the primitive matter has collected in distinct, planetary masses; while a similar result has been prevented in the Saturnian rings by their proximity to the central body. In one respect, however, we observe a striking correspondence. It has been shown that several positions occur in the asteroid zone where planetary periods would have simple relations of commensurability with the period of Jupiter, and that portions of the original ring occupying these positions would be liable to great disturbance. Now, the ring of Saturn is evidently subject to like perturbation by the nearest satellites. Hence gaps or chasms, analogous to those in the zone of asteroids, ought also to be found in the secondary ring. It has accordingly been noticed that Cassini's, or rather Ball's division occurs precisely where the periods of satellites would be commensurable with those of the four members of the system immediately exterior.*

But astronomers have sometimes seen the ring of Saturn apparently separated by several black lines into concentric annuli. At other times, however, no such divisions could be detected. The fact, therefore, of the permanence of these gaps is extremely doubtful, except in the case of a division of the exterior bright ring. This has been frequently seen by eminent astronomers; and it is probable, though not absolutely certain, that it is never entirely closed. Most observers agree in placing it outside of the middle of the exterior ring. Let us now inquire whether any simple relation of commensurability obtains between the periods of satellites revolving at the distance of this outermost gap, and those of Mimas, Enceladus, Tethys, and Dione.

$\frac{4}{7}$ of the period of Mimas	=	$12^h\ 56^m$
$\frac{2}{7}$ " Tethys	=	12 57

* *Meteoric Astronomy*, Chap. XII.

$\frac{1}{5}$	"	Dione	= 13	8
$\frac{2}{5}$	"	Enceladus	= 13	9

The interior radius of the outer ring = 1.9963

The radius of a circle bisecting the outer ring = 2.1209

The distance of a satellite whose period is $12^h\ 56^m$ = 2.1473

The distance of a satellite whose period is $13^h\ 9^m$ = 2.1510

The exterior radius of the outer ring = 2.2456

It is thus seen that just beyond the middle of the outer ring, where the division is actually found, another zone occurs in which the periodic times of satellites would be commensurable with those of Mimas, Enceladus, Tethys and Dione.

The FACTS detailed in the preceding pages are unquestionable. In regard to the proposed *explanation* of these facts the writer would speak with becoming caution. In his humble attempt to reduce a large class of isolated truths to the domain of law some important considerations may have been overlooked. Be this as it may, he indulges the hope that abler astronomers may deem the enquiry not unworthy their researches.

Stated Meeting, Oct. 6th., 1871.

Present, fifteen members.

Vice President, Mr. FRALEY, in the chair.

A photograph for the Album was received from Professor E. N. Horsford, dated Cambridge, Massachusetts, October 29.

Letters of acknowledgment were read from the London Meteorological Office, September 22 (83, 84, 85); and the Buffalo Society of Natural Sciences, December 1, 1870 (XI Pro).

Letters of envoi were received from the Natural History and Historical Union of Donauerschingen, September 15, and the United States Secretary of the Interior, Washington, D. C., September 15, 1871.

A recent letter from Mr. Carlier to Mr Durand, was read by Mr. Price, who offered a Resolution, which was adopted, authorizing the presiding officer of the meeting to execute